$\cdots \to \mathbf{0} \to \mathbb{Z}^{\mathsf{rx}} \xrightarrow[\mathsf{Sx}]{\partial_2} \mathbb{Z}^{\mathsf{N}} \xrightarrow[\mathsf{Hz}]{\partial_1} \widehat{\mathbb{T}}^{\mathsf{rz}} \cong \mathbb{Z}^{\mathsf{rz}} \to \mathbf{0} \to \cdots$

$\operatorname{coker}\left(R^m \xrightarrow{A} R^n\right) \cong \bigoplus_i R/\alpha_i R$

Quantum error correction with rotors and torsion

Samo Novák
COSMIQ
Inria Paris

Journées C^2
April 2025

Inria

Rotor

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A quantum **rotor** is a system whose states we describe using the **circle** group

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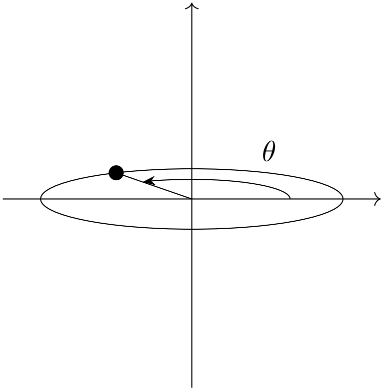

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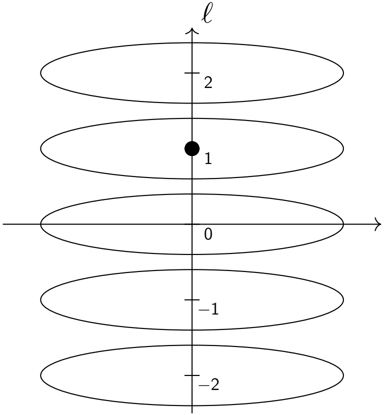
(We won't worry about the duality today.)



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From now on, we **focus on one** of these spaces: the one **described by** $\mathbb{Z}$. Results on Pontryagin duality allow this (but we won't go into the details).

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Conclusion: need **more general** machinery, unexpected phenomena happen: **torsion**.

Okay whatever. Why is this REALLY interesting?

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Motivation: Revealing subtleties

The more general machinery reveals **subtle details** that are also present, but **hidden**, in more **conventional** constructions.

Escape-velocity intro to module theory

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Idea

If R is **not a field**, sometimes **weird stuff** happens.

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Example

Let $M = \mathbb{Z}_2 = {}^{\mathbb{Z}}\!/_{2\mathbb{Z}} = \{\bar{0}, \bar{1}\}$ as $\mathbb{Z}$-module. Then:

$$4 \cdot \bar{1} = \overline{4 \cdot 1 \pmod 2} = \bar{0}.$$

The whole $\mathbb{Z}_2$ is a torsion $\mathbb{Z}$-module!

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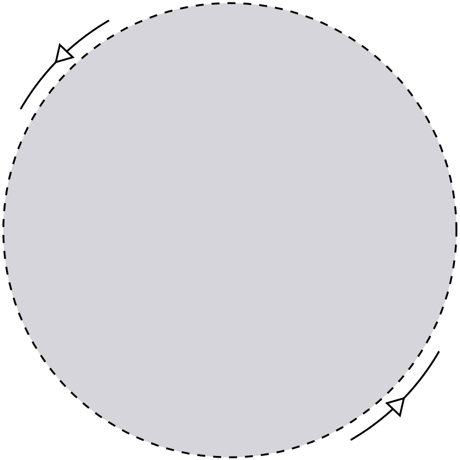

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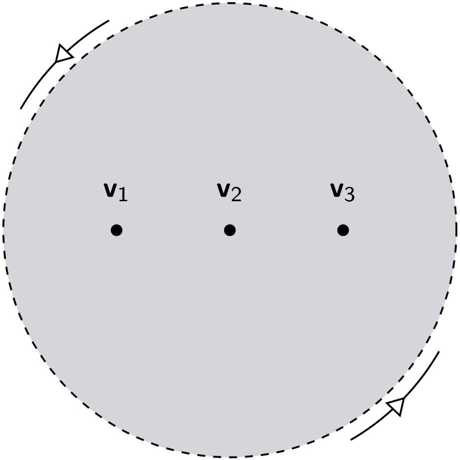

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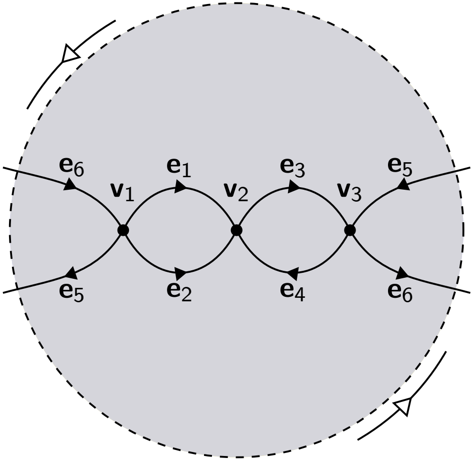

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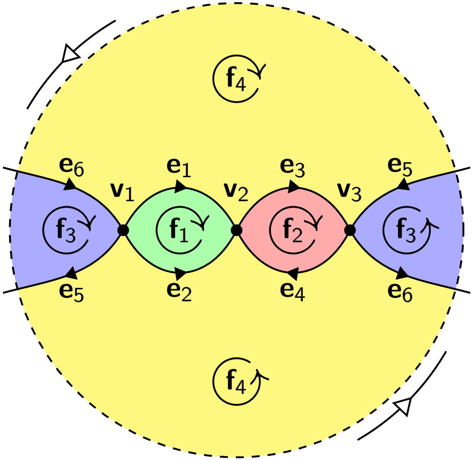

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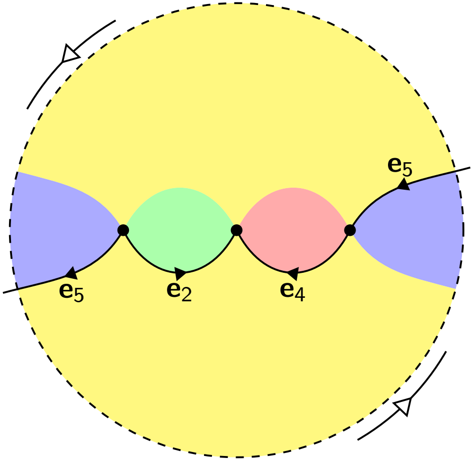

Intuition

Torsion, here $\mathbb{Z}_2$, corresponds to a **cycle** which must be **traversed multiple times** (2) to **come back** exactly to the **same point** in the **same orientation**.

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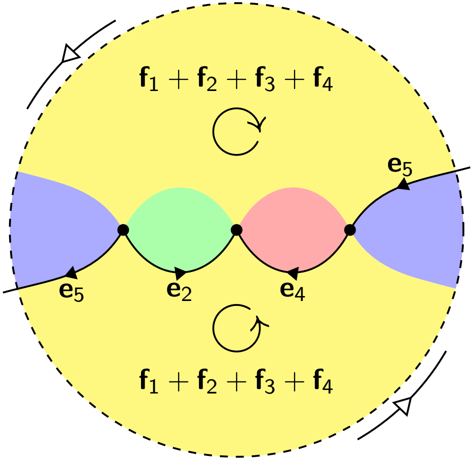

Note *linear* structure!

More precisely

Cycle $\underline{x} = -\mathbf{e}_2 + \mathbf{e}_4 + \mathbf{e}_5$ is **not a boundary** of a face. But its **multiple** $2\underline{x}$ is the boundary of $\mathbf{f}_1 + \mathbf{f}_2 + \mathbf{f}_3 + \mathbf{f}_4$.
Thus $\mathbb{Z}_2$ **torsion**.

Chain complex: the previous idea in more detail

A way to construct a code is to **cellulate** a topological **space** and build a **chain complex**:

Definition (chain complex CSS code)

A **chain complex** representing a **CSS code** is:

$$C_2 = \mathbb{Z}^{r_X} \xrightarrow{\partial_2 = H_X^\top} C_1 = \mathbb{Z}^N \xrightarrow{\partial_1 = H_Z} C_0 = \mathbb{Z}^{r_Z}$$

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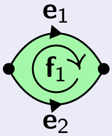

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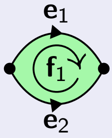

These boundary maps give **parity check matrices** H_X and H_Z.

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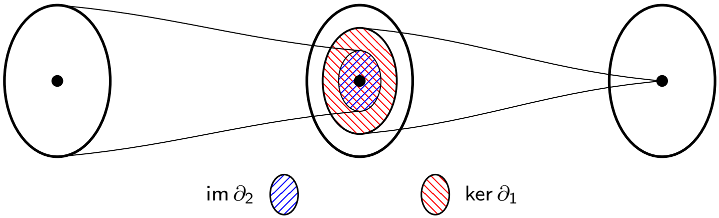

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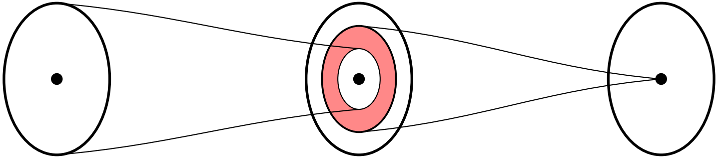

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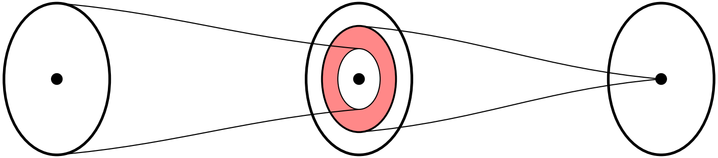

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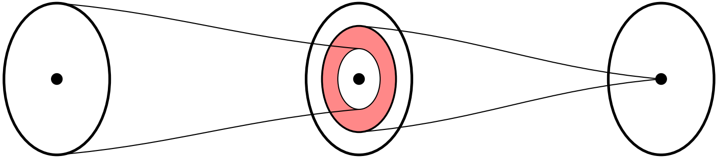

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Key idea

Torsion comes from a **cycle** of edges that is **not a boundary**, but its **multiple is**.

This is given by the **image** $\operatorname{im} \partial_2$.

What does torsion mean?

In the $\mathbb{RP}^2$ example, the **homology** is $H_1(\mathbb{RP}^2) = \mathbb{Z}_2$, even though it is constructed from rotors ($\mathbb{Z}$). This is **torsion**.

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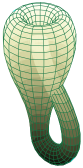

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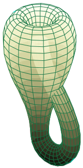
$$H_1(\mathbb{K}^2) = \mathbb{Z} \oplus \mathbb{Z}_2.$$

A rotor code defined on a cellulation of $\mathbb{K}^2$ encodes a **rotor** and a **qubit**.



[Ttt06]

Key observation

We can obtain **mixed-dimension** systems.

$\cdots \to \mathbf{0} \to \mathbb{Z}^{r_x} \xrightarrow[S_x]{\partial_2} \mathbb{Z}^N \xrightarrow[H_z]{\partial_1} \widehat{\mathbb{T}}^{r_z} \cong \mathbb{Z}^{r_z} \to \mathbf{0} \to \cdots$ $\quad \operatorname{coker}(\mathbf{\cdot} \xrightarrow{A} R^n) \cong \bigoplus_i R/\alpha_i R$

Thank you for your attention!

Quantum error correction with rotors and torsion

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Bibliography follows.

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